VIBRATION ANALYSIS ON PLATES BY ORTHOGONAL POLYNOMIALS

Byoung Kee Han*, Kang Chung and Dae Sik Han****

(Received March 20, 1989)

This paper presents vibration analysis of plates by the Rayleigh-Ritz method with orthogonal polynomials derived by the Gram-Schmidt Process as displacement functions, and Gauss-Legendre Quadrature as an integration scheme. A computer program was developed and numerical results by this computation were in good accord with those obtained by using other beam functions. Furthermore, the present method was shown to resolve various problems encountered in the application of existing methods.

Key Words: Orthogonal Polynomials, Gram-Schmidt Process, Gauss-Legendre Quadrature, Orthogonality Properties, Beam Function.

NOMENCLATURE

a, b : Length scale of rectangular plate in x and y directions, respectively
C : Clamped edge indicater
D : Flexural rigidity, $Eh^3/\{12(1-\nu^2)\}$
E : Young's modulus
F : Free edge indicator
S : Simply-supported edge indicator
T_{max} : Maximum total kinetic energy
U_{max} : Maximum total strain energy
$X_m(x)$, $Y_n(y)$: Orthogonal set of polynomials
α : Aspect ratio
$\phi(x)$: Orthogonal polynomial
ω : Circular frequency
λ : Frequency parameter
ν : Poisson's ratio
ρ : Mass density per unit area of plate

1. INTRODUCTION

Plates are important structural components extensively used such as in bridges, ship deck-plates, railroads and aircraft structures. For reliable design and safe use, it is essential to assess dynamic properties of such plates. Therefore, a number of studies on vibration characteristics of plates have been carried out by using the Rayleigh or Rayleigh-Ritz method(Young, 1950; Warburton, 1954; Hearmon et al., 1959), the Galerkin method (Munakata, 1952; Stanisic et al., 1957), the finite difference method (Hidaka, 1951; Nishimura, 1953; Abramowitz et al., 1955) and other numerical methods (Cheung, 1971; Hooker et al., 1974). In the application of these methods, the suitable selection of admissible functions is a key for obtaining accurate results. Dickinson(1978) and Mizusawa(1986) employed a simply supported(S.S.) function and B-spline function respectively in the Rayleigh-Ritz method.

These functions proposed in the past studies, however, have limitations in general application. For instance, the S.S. function yields results in good agreement with those obtained by beam function for plates with two parallel edges simply supported. But the solutions become less accurate for plates with one or more free edges. In methods by finite elements, error generally may occur by degree of discretization. Trigonometric beam functions often induce complexity in integration.

In this study, a computer program was developed to solve the above-mentioned problems and to simplify the vibration analysis.

This computation employs orthogonal polynomials obtained by the Gram-Schmidt process as displacement functions in the Rayleigh-Ritz method. Integration was done with Gauss-Legendre quadrature for all possible boundary conditions in vibration of plates. To verify validity of the present method, computed results were compared with those by Leissa(1973) and Dickinson(1982).

2. METHOD OF ANALYSIS

2.1 Application of Rayleigh-Ritz Method

A rectangular plate as shown in Fig. 1 is used to analyze vibration characteristics. The assumed transverse displacement function $W(x, y)$ is given by

$$W(x, y)=\sum_m\sum_n A_{mn}X_m(x)\,Y_n(y) \quad (1)$$

where $X_m(x)$ and $Y_n(y)$ are functions of characteristic orthogonal polynomials, and x and y represent the normalized directions, $x=\xi/a$, $y=\eta/b$ respectively.

The maximum potential energy U_{max} stored in the stressed

*Department of Mechanical Engineering, Hongik University, Seoul 121-791, Korea
**Department of Mechanical Engineering, Hongik Graduate School, Seoul 121-791, Korea

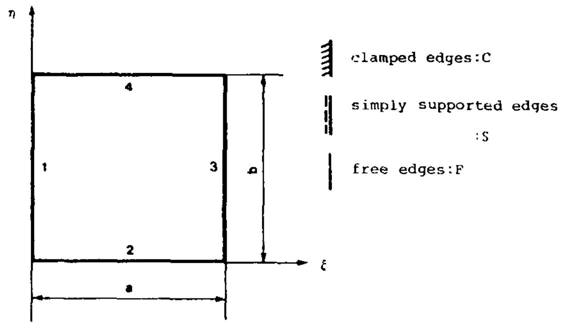


Fig. 1 Rectangular plate and symbols for boundary conditions

elastic body is given by Love's theory as follows.

$$U_{max}=\frac{1}{2}Dab\int_0^1\int_0^1\Big[\Big(\frac{\partial^2 w}{\partial x^2}\Big)^2+\alpha^4\Big(\frac{\partial^2 w}{\partial y^2}\Big)^2 +2\nu\alpha^2\frac{\partial^2 w\partial^2 w}{\partial x^2\partial y^2}+2(1-\nu)\alpha^2\Big(\frac{\partial^2 w}{\partial x\partial y}\Big)^2\Big]dxdy \quad (2)$$

And the maximum kinetic energy T_{max} is represented by

$$T_{max}=\frac{1}{2}\rho hab\omega^2\int_0^1\int_0^1 W(x,y)\,dxdy \quad (3)$$

where D is flexural rigidity of the plate defined as $Eh^3/\{12(1-\nu^2)\}$, ν is Poisson's ratio, ρ is density, h is thickness of the plate and α is aspect ratio($=a/b$).

From the minimum total energy principle,

$$\frac{\partial U}{\partial A_{kl}}-\frac{\partial T}{\partial A_{kl}}=0 \quad (4)$$

where $k=1,2,\cdots m$: $l=1,2,\cdots n$

Substitution of deflection function Eq.(1) into energy expressions Eq.(2) and Eq.(3), and solving Eq.(4) yields the following equation

$$D\sum_m\sum_n A_{mn}\int_0^1\int_0^1[(X_m''X_k''Y_nY_l) +\alpha^4(X_mX_kY_n''Y_l'') +\nu\alpha^2(X_m''X_kY_nY''_l+X_mX_k''Y_n''Y_l) +2(1-\nu)\alpha^2(X_m'X_k'Y_n'Y_l')]dxdy -\rho h\omega^2\sum_m\sum_n A_{mn}\int_0^1\int_0^1(X_mX_kY_nY_l)\,dxdy=0 \quad (5)$$

Rearranging Eq.(5) yields eigenvalue equation

$$\sum_m\sum_n[B_{mn}^{(kl)}-\lambda\bar{B}_{mn}^{(kl)}]A_{mn}=0 \quad (6)$$

where

$$B_{mn}^{(kl)}=\int_0^1\int_0^1\Big[(X_m''X_k''Y_nY_l) +\alpha^4(X_mX_kY_n''Y_l'') +\nu\alpha^2(X_m''X_kY_nY_l''+X_mX_k''Y_n''Y_l)) +2(1-\nu)\alpha^2(X_m'X_k'Y_n'Y_l')]dxdy \quad (7\text{-a})$$

$$\bar{B}_{mn}^{(kl)}=\int_0^1\int_0^1(X_mX_kY_nY_l)\,dxdy \quad (7\text{-b})$$

$m,n,k,l=1,2,3,\cdots$ and $\lambda=\rho h\omega^2a^4/D$

2.2 Orthogonal Polynomials by Gram-Schmidt Process

In the interval $[a,b]$, polynomial functions $\{\phi_0,\phi_1,\phi_2,\cdots,\phi_k,\cdots,\phi_n\}$ for weight function w are defined as

$$\phi_0(X) : \text{function satisfying orghogonality} \quad (8)$$
$$\phi_1(X)=(X-B_1)\phi_0(X) \quad (9)$$

where

$$B_1=\int_0^1 xw(x)[\phi_0(x)]^2dx\Big/\int_0^1 w(x)[\phi_0(x)]^2dx \quad (10)$$

When $k\geqq 2$,

$$\phi_k(x)=(x-B_k)\phi_{k-1}(x)-C_k\phi_{k-2}(x) \quad (11)$$

where

$$B_k=\int_0^1 xw(x)\phi_{k-1}^2(x)\,dx\Big/\int_0^1 w(x)\phi_{k-1}^2(x)\,dx \quad (12)$$

$$C_k=\int_0^1 xw(x)\phi_{k-1}(x)\phi_{k-2}(x)\,dx\Big/\int_0^1 w(x)\phi_{k-2}^2(x)\,dx \quad (13)$$

In this study, weight function $w(x)$ was chosen as 1. Since polynomial function $\phi_k(x)$ safisfy orthorgonality condition, it can be written as

$$\int_0^1 w(x)\phi_k(x)\phi_l(x)\,dx=\begin{cases}0 & \text{if } k\neq l\\ a_{kl} & \text{if } k=l\end{cases} \quad (14)$$

To be noted is that polynomial function $\phi_k(x)$ also satisfies all boundary conditions, as beam functions do.

2.3 Orthogonal Polynomials for Various Boundary Conditions

(1) Clamped-Clamped boundary conditions(C-C)

Boundary condition equations in C-C beam are

$$X(0)=0 \qquad X(1)=0$$
$$X'(0)=0 \qquad X'(1)=0 \quad (15)$$

and a function satisfying all geometric boundary conditions of beam can be written as

$$X(x)=P_0+P_1x+P_2x^2+P_3x^3+P_4x^4 \quad (16)$$

By applying Eq. (15) to Eq. (16),

$$X(x)=P_4(x^2-2x^3+x^4) \quad (17)$$

where P_4 is an arbitrary constant.

Then, the normalized orthogonal polynomial is

$$\phi_0(x)=(x^2-2x^3+x^4)\Big/\Big[\int_0^1 X^2(x)\,dx\Big]^{\frac{1}{2}} \quad (18)$$

(2) Simply supported-simply supported boundary conditions(S-S)

Boundary condition equations in S-S beam are

$$X(0)=0 \qquad X(1)=0$$
$$X''(0)=0 \qquad X''(1)=0 \quad (19)$$

and a function satisfying all geometric boundary conditions of beam can be written as

$$X(x)=P_0+P_1x+P_2x^2+P_3x^3+P_4x^4 \tag{20}$$

By applying Eq. (19) to Eq. (20)

$$X(x)=P_4(x-2x^3+x^4) \tag{21}$$

where P_4 is an arbitrary constant.

Then, the normalized orthogonal polynomial is

$$\phi_0(x)=(x-2x^3+x^4)\Big/\left[\int_0^1 X^2(x)\,dx\right]^{\frac{1}{2}} \tag{22}$$

(3) Free-Free boundary conditions(F-F)

Boundary condition equations in F-F beam a e

$$X''(0)=0 \quad X''(1)=0$$
$$X'''(0)=0 \quad X'''(1)=0 \tag{23}$$

and a function satisfying all geometric boundary conditions of beam can be written as

$$X(x)=P_0+P_1x+P_2x^2+P_3x^3+P_4x^4+P_5x^5+P_6x^6 \tag{24}$$

By applying Eq. (23) to Eq. (24),

$$X(x)=P_5\left(C_0+C_1x-\frac{5}{6}x^4+x^5-\frac{1}{3}x^6\right) \tag{25}$$

where P_5 is an arbitrary constant.

Then, the normalized orthogonal polynomial is

$$\phi_0(x)=\left(C_0+C_1x-\frac{5}{6}x^4+x^5-\frac{1}{3}x^6\right)\Big/\left[\int_0^1 X^2(x)\,dx\right]^{\frac{1}{2}} \tag{26}$$

(4) Clamped-Free boundary conditions(C-F)

Boundary condition equations in C-F beam are

$$X(0)=0 \quad X''(1)=0$$
$$X'(0)=0 \quad X'''(1)=0 \tag{27}$$

and a function satisfying all geometric boundary conditions of beam can be written as

$$X(x)=P_0+P_1x+P_2x^2+P_3x^3+P_4x^4 \tag{28}$$

By applying Eq. (27) to Eq. (28),

$$X(x)=P_4(6x^2-4x^3+x^4) \tag{29}$$

where P_4 is an arbitrary constant.

Then, the normalized orthogonal polynomial is

$$\phi_0(x)=(6x^2-4x^3+x^4)\Big/\left[\int_0^1 X^2(x)\,dx\right]^{\frac{1}{2}} \tag{30}$$

(5) Cl.mped-Simply supported boundary conditions(C-S)

Boundary condition equations in C-S beam are

$$X(0)=0 \quad X(1)=0$$
$$X'(0)=0 \quad X''(1)=0 \tag{31}$$

and a function satisfying all geometric boundary conditions of beam can be written as

$$X(x)=P_0+P_1x+P_2x^2+P_3x^3+P_4x^4 \tag{32}$$

By applying Eq. (31) to Eq. (32),

$$X(x)=P_4\left(\frac{3}{2}x^2-\frac{5}{2}x^3+x^4\right) \tag{33}$$

where P_4 is an arbitrary constant.

Then, the normalized orthogonal polynomial is

Table 1 Constants B_k and C_k for various boundary conditions

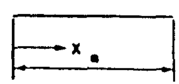
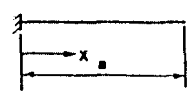
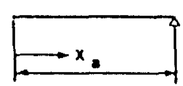
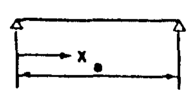
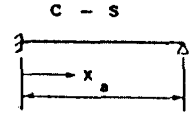
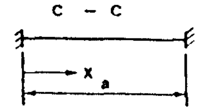
Boundary conditions	B_k : $k=1,2,3,\cdots$	C_k : $k=2,3,4,\cdots$
F - F 	0.501190172	0.149998348
	0.498828559	0.028571876
	0.501134700	0.099204330
	0.498875039	0.040405048
	0.501114267	0.085661817
	0.498890421	0.046155425
	0.501104575	0.079408668
C - F 	0.802197802	0.024935218
	0.645534817	0.038924810
	0.588591081	0.046495104
	0.559995589	0.050964562
	0.543421003	0.053805922
	0.532867133	0.109440559
	0.533546326	0.054313099
F - S 	0.750249364	0.037453653
	0.583432525	0.050781332
	0.541693154	0.055798890
	0.525011258	0.058179497
	0.516672864	0.059487804
	0.511908213	0.060281739
	0.508930975	0.060799079
S - S 	0.500000000	0.032991202
	0.500000000	0.046068627
	0.500000000	0.052169770
	0.500000000	0.055432306
	0.500000000	0.057366502
	0.500000000	0.058604223
	0.500000000	0.059443187
C - S 	0.565789477	0.026772224
	0.547673218	0.039497719
	0.535329104	0.046447212
	0.526948072	0.500650506
	0.521128882	0.053388693
	0.516919158	0.055273600
	0.513908921	0.056627263
C - C 	0.499999876	0.022727269
	0.500000238	0.034965034
	0.499999762	0.042307690
	0.500000238	0.047058831
	0.499999762	0.050309600
	0.500000238	0.052631565
	0.499999762	0.054347830

Table 2 Orthogonal polynomials for various boundary conditions

Boundary conditions		Function
C—C		$\phi_0(x)=\dfrac{(x^2-2x^3+x^4)}{0.039840959P_4}$
S—S		$\phi_0(x)=\dfrac{(x-2x^3+x^4)}{0.221825041P_4}$
F—F		$\phi_0(x)=\dfrac{\left(C_0+C_1x-\frac{5}{6}x^4+x^5-\frac{1}{3}x^6\right)}{192.4985805P_5}$
C—F		$\phi_0(x)=\dfrac{6x^2-4x^3+x^4)}{1.5202339P_4}$
C—S		$\phi_0(x)=\dfrac{\left(\frac{3}{2}x^2-\frac{5}{2}x^3+x^4\right)}{0.086831348P_4}$
S—F		$\phi_0(x)=\dfrac{\left(C_2+\frac{10}{3}x^3-\frac{10}{3}x^4+x^5\right)}{192.8900174P_4}$

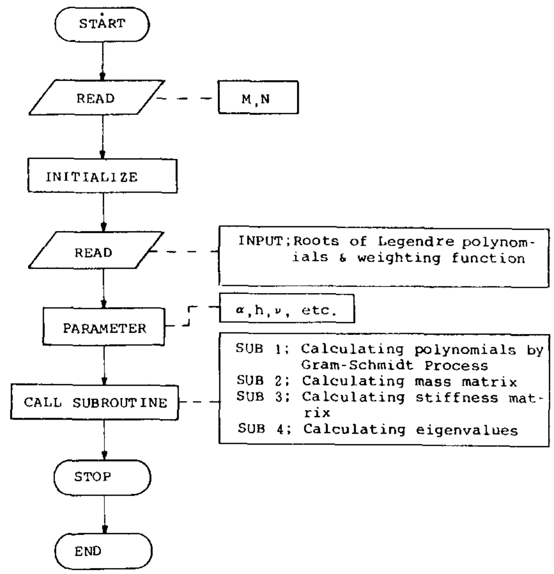


Fig. 2 Flow chart of main program(polynomials by Gram-Schmidt process)

$$\phi_0(x)=\left(\frac{3}{2}x^2-\frac{5}{2}x^3+x^4\right)\Big/\left[\int_0^1 X^2(x)\,dx\right]^{\frac{1}{2}} \tag{34}$$

(6) Simply supported-Free boundary conditions(S-F)
Boundary condition equations in S-F beam are

$$X(0)=0 \qquad X''(1)=0$$
$$X''(0)=0 \qquad X'''(1)=0 \tag{35}$$

and a function satisfying all geometric boundary conditions of beam can be written as

$$X(x)=P_0+P_1x+P_2x^2+P_3x^3+P_4x^4+P_5x^5 \tag{36}$$

By applying Eq. (35) to Eq. (36),

$$X(x)=P_5\left(C_2x+\frac{10}{3}x^3-\frac{10}{3}x^4+x^5\right) \tag{37}$$

where P_5 is an arbitrary constant.
Then, the normalized orthogonal polynomial is

$$\phi_0(x)=\left(C_2x+\frac{10}{3}x^3-\frac{10}{3}x^4+x^5\right)\Big/\left[\int_0^1 X^2(x)\,dx\right]^{\frac{1}{2}} \tag{38}$$

Here, the functions $\phi_0(x)$ for the free-free and the simply supported-free boundary conditions satisfy the geometric boundary conditions, regardless of the arbitrary values of constants C_0, C_1 and C_2. But the natural frequencies of a plate depend on the values of the constants.

In this study, constants C_0, C_1 and C_2 were determined as 1000/3, −2000/3 and 1000/3, respectively, by selecting among values obtained to repetitively compute for several values. The functions $Y(y)$ are determined in the same manner, by substituting y into x in the above equations. Table 1 and 2 list the values of the constants (B_k and C_k) and orthogonal polynomials obtained for various boundary conditions, respectively.

3. ANALYSIS RESULTS AND DISCUSSIONS

3.1 Structures of Computer Program

The computer program developed in this study was based on numerical analysis developed by using Gram-Shmidt process to obtain proper functions for given boundary conditions of plates and Gauss-Legendre integration method. Compared to the conventional Simpson formula applicable only to functions with limited orders, Gauss-Legendre quadrature can be used even for irregular functions.

The main computation steps of this program include:

(1) Input date, i.e, weighting factors and roots of Legendre polynomials for integration of functions satisfying given boundary counditions.

(2) Read-in test parameters, i.e, aspect ration (α), Poisson's ratio (ν) and desired number of eigenvalues, etc.

(3) Obtain polynomials by Gram-Schmidt process.

(4) Compute mass matirx.

(5) Compute stiffness matrix.

(6) Analyze mass matrix and stiffness matrix and compute eigen problems.

Figure 2 shows the flow chart of main program. The program written in FORTRAN-77 and computations were performed on MV-8000, IBM-PC XT and AT computers at Hongik University.

3.2 Numerical Results and Discussions

Since the plate in this study was assumed to have thickness much smaller than lengths in ξ and η directions. Stresses in the thickness direction was neglected. Further, by assuming the absence of in-plane force, frequency parameters $\lambda(=\rho h\omega^2a^4/D)$ were obtained for free vibration due to bending only. Numerical calculations were performed for aspect ratio α of 0.4, 1.0 and 2.5. Poisson's ratio ν of 0.3 was used. The number of eigenvalues were determined by the functions $\phi_k(x)$ with k of larger than 5. The computed results for matrix sizes of more than 25×25 or 36×36 were compared with data by Leissa and Dickinson up to the 6th degree of mode.

The Gauss-Legendre quadrature method used in the present program removes difficulties in integration of trigonometric functions involved in beam functions. As a result, vibration analysis by computers become more handy and flexible. As previously pointed out, the S.S. plate functions suggested by Dickinson are erroneous for plates with one or more free edges, although they produce natural frequencies in agreement with those obtained by using beam functions for plates with simply supported edges. The present orthogonal polynomials by the Gram-Schmidt porcess eliminate this problem.

Frequency parameters of plates are listed in Table 3 to 5 for zero, 6 to 7 for one, 8 to 10 for two and 11 for than three free edges. Values of frequency parameters are quite satisfactory for all permissible boundary conditions in plates, with an accuracy of 1 to 2% for zero free-edge and 2 to 3% for more than free-edges. These results demonstrated that orthogonal

Table 3 Frequency parameters $\sqrt{\lambda}=\omega a^2\sqrt{\rho/D}$

Boundary conditions	Mode No.	Aspect ratio $\alpha=a/b$(Leissa/present[Dickinson])		
		0.4	1.0	2.5
CCCC	1.	23.648/23.644	35.992/ 35.986[35.988]	147.80/147.774[147.799]
	2.	27.817/27.808	73.413/ 73.393[73.406]	173.85/173.798[173.839]
	3.	35.446/35.418	73.413/ 73.393[73.406]	221.54/221.35 [221.49]
	4.	46.702/46.805	108.27 /108.22 [108.25]	291.89/292.53 [291.83]
	5.	61.554/62.383	131.64 /131.78 [131.62]	384.71/389.89 [384.61]
	6.	63.100/63.083	132.24 /132.41 [132.23]	394.37/394.27 [394.37]

Table 4 Frequency parameters $\sqrt{\lambda}=\omega a^2\sqrt{\rho/D}$

Boundary conditions	Mode No.	Aspect ratio $\alpha=a/b$(Leissa/present)		
		0.4	1.0	2.5
SSSS	1.	11.4487/11.4487	19.7392/ 19.7392	71.5564/ 71.5547
	2.	16.1862/16.1862	49.3480/ 49.3481	101.1634/101.1635
	3.	24.0818/24.1586	49.3480/ 49.3480	150.5115/150.9912
	4.	35.1358/35.6669	78.9568/ 78.9569	219.5987/222.9184
	5.	45.0576 /45.0576	98.6960/ 99.3042	256.6097/256.6102
	6.	45.7950/45.7950	98.6960/ 99.3042	286.2185/286.2190
CCSS	1.	16.849 /16.848	27.056 / 27.054	105.31 /105.30
	2.	21.368 /21.358	60.544 / 60.539	133.50 /133.49
	3.	29.236 /29.257	60.791 / 60.787	182.73 /182.86
	4.	40.509 /40.930	92.865 / 92.838	253.18 /255.81
	5.	51.457 /51.452	114.57 /114.85	321.60 /321.57
	6.	55.117 /55.964	114.72 /114.99	344.48 /349.78
SCSC	1.	12.1347/12.1347	28.9509/ 28.9509	145.4839 /145.4839
	2.	18.3647/18.3647	54.7431/ 54.7432	164.7387/164.7387
	3.	27.9657/27.9661	69.3270/ 69.3270	202.2271/202.2301
	4.	40.7500/40.8996	94.5850/ 94.9853	261.1053/263.9788
	5.	41.3782/41.3783	102.2162/102.8070	342.1442/353.7434
	6.	47.0009/47.0023	129.0955/129.2993	392.8746/392.8749

Table 5 Frequency parameters $\sqrt{\lambda}=\omega a^2\sqrt{\rho/D}$

Boundary conditions	Mode No.	Aspect ratio $\alpha=a/b$(Leissa/present)		
		0.4	1.0	2.5
SCSS	1.	11.7502/11.7503	23.6463/ 23.6463	103.9227/103.9227
	2.	17.1872/17.1873	51.6743/ 51.6744	128.3382/128.3383
	3.	25.9171/25.9515	58.6464/ 58.6465	172.3804/172.8091
	4.	37.8317/38.2932	86.1345/ 86.1348	237.2502/240.3741
	5.	41.2070/41.2079	100.2698/100.8704	320.7921/320.7937
	6.	46.3620/46.3652	113.2281/113.5220	322.9642/322.7389
CCCS	1.	23.440 /23.439	31.829/ 31.826	107.07 /107.04
	2.	27.022 /27.017	63.347 / 63.331	139.66 /139.61
	3.	33.799 /33.387	71.084 / 71.077	194.41 /194.41
	4.	44.131 /44.300	100.83 /100.79	370.48 /271.23
	5.	58.034 /59.687	116.40 /116.64	322.55 /322.46
	6.	62.971 /62.965	130.37 /130.55	353.43 /353.17

Table 6 Frequency parameters $\sqrt{\lambda}=\omega a^2\sqrt{\rho/D}$

Boundary conditions	Mode No.	Aspect ratio $\alpha=a/b$(Leissa/present)		
		0.4	1.0	2.5
CCCF	1.	22.577 /22.531	24.020 / 23.938	37.656/ 37.588
	2.	24.623 /24.597	40.039 / 40.009	76.407/ 76.138
	3.	29.244 /29.245	63.493 / 63.253	135.15 /134.79
	4.	37.059 /37.990	76.761 / 76.834	152.47 /152.37
	5.	48.283 /48.624	80.731 / 80.595	193.01 /192.78
	6.	61.922 /61.790	116.80 /116.80	213.74 /213.95
SSSF	1.	10.1259/10.1262	11.6845/ 11.6808	18.8009/ 18.7869
	2.	13.0570/13.0567	27.7563/ 27.7436	50.5405/ 50.5192
	3.	18.8390/18.9460	41.1967/ 41.1940	100/2321/100.8088
	4.	27.5580/28.9628	59.0655/ 59.0561	110.2259/110.1411
	5.	39.3377/39.6397	61.8606/ 62.4031	147.6317/147.5515
	6.	39.6118/39.7200	90.2941/ 90.9457	169.1026/172.8254
SCSF	1.	10.1888/10.1894	12.6874/ 12.6874	30.6277/ 30.6277
	2.	13.6036/13.6042	33.0651/ 33.0651	58.0804/ 58.0805
	3.	20.0971/20.1209	14.7019/ 41.7030	105.5470/106.1231
	4.	29.6219/30.7874	63.0148/ 63.0160	149.4569/149.4570
	5.	39.6382/39.6572	72.3976/ 71.5156	173.1060/176.7855
	6.	42.2425/43.0119	90.6114/ 91.2599	182.8110/182.8110

Table 7 Frequency parameters $\sqrt{\lambda}=\omega a^2\sqrt{\rho/D}$

Boundary conditions	Mode No.	Aspect ratio $\alpha=a/b$(Leissa/present)		
		0.4	1.0	2.5
CCSF	1.	15.696/15.638	17.615/ 17.543	33.578/ 33.534
	2.	18.373/18.344	36.046/ 36.027	66.612 / 66.383
	3.	23.987/23.965	52.065/ 51.828	119.90 /119.64
	4.	32.810/32.814	71.194/ 71.089	150.83 /150.79
	5.	44.862/44.261	74.349/ 74.445	187.61 /187.47
	6.	50.251/50.088	106.28 /106.14	193.23 /195.52
CSCF	1.	22.544/22.497	23.460/ 23.385	28.564/ 28.478
	2.	24.296/24.271	35.612/ 35.575	70.561/ 70.289
	3.	28.341/28.321	63.126/ 62.906	114.00 /113.82
	4.	35.345/35.736	66.808/ 67.292	130.83 /130.51
	5.	45.710/45.972	77.502/ 77.394	159.54 /159.25
	6.	59.562/61.760	108.99 /109.47	210.32 /210.60
CSSF	1.	15.649/15.596	16.865/16.795	23.067/ 23.000
	2.	17.946/17.919	31.138/ 31.107	59.969/ 59.724
	3.	22.902/22.882	51.631/ 51.410	111.95 /111.83
	4.	30.892 /31.332	64.043/ 64.559	115.11 /114.87
	5.	42.108/42.542	67.646/ 67.545	153.24 /153.02
	6.	50.222/50.073	101.21 /101.73	189.49 /191.88

Table 8 Frequency parameters $\sqrt{\lambda}=\omega a^2\sqrt{\rho/D}$

Boundary conditions	Mode No.	Aspect ratio $\alpha=a/b$(Leissa/present)		
		0.4	1.0	2.5
SSFF	1.	1.3201/ 1.3168	3.3687/ 3.3600/[3.6991]	8.251/ 8.230[10.807]
	2.	4.7433/ 4.7250	17.407 /17.298 /[17.334]	29.646/ 29.531[30.130]
	3.	10.362 /10.404	19.367 /19.273 /[19.393]	64.760/ 65.023[64.613]
	4.	15.873 /15.783	38.291 /38.185 /[38.256]	99.206/ 89.643/[99.249]
	5.	18.930 /19.542	51.324 /51.514 /[51.249]	118.31 /122.14 /[117.95]
	6.	20.171 /20.481	53.738 /54.033 /[53.677]	126.07 /128.01 [126.09]
CCFF	1.	3.9857/ 3.7968	6.9421/ 6.9268/[7.1631]	24.911/ 24.855[26.039]
	2.	7.1551/ 7.1442	24.034 /23.9428/[23.974]	44.719/ 44.651[45.081]
	3.	13.101 /13.306	26.681 /26.610 /[26.687]	81.879/ 83.166[81.730]
	4.	21.844 /22.212	47.785 /47.755 /[47.753]	136.52 /138.826[136.24]
	5.	22.896 /23.717	63.039 /63.918 /[62.967]	143.10 /148.23 [142.99]
	6.	26.501 /26.519	65.833 /67.079 /[65.772]	165.63 /165.75 [165.64]

Table 9 Frequency parameters $\sqrt{\lambda}=\omega a^2\sqrt{\rho/D}$

Boundary conditions	Mode No.	Aspect ratio $\alpha=a/b$(Leissa/present)		
		0.4	1.0	2.5
CSFF	1.	3.8542/ 3.8450	5.3639/ 5.3500	10.100/ 10.082
	2.	6.4198/ 6.4033	19.171 /19.067	35.157 / 35.037
	3.	11.578 /11.616	24.768 /24.673	74.990/ 74.779
	4.	19.767 /20.623	43.191 /43.087	99.928/ 99.319
	5.	22.521 /22.563	53.000 /54.214	127.69 /127.48
	6.	26.024 /26.024	64.050 /64.913	135.45 /141.49
CFSF	1.	15.382 /15.367	15.285 /15.260	15.128 / 15.133
	2.	16.371 /16.305	20.673 /20.598	37.294/ 37.242
	3.	19.656 /19.342	39.775 /40.345	49.226/ 50.963
	4.	25.549 /26.153	49.730 /50.485	83.325/ 83.077
	5.	34.507 /35.229	56.617 /56.321	103.14 /103.44
	6.	46.435 /46.901	77.368 /78.358	143.68 /143.28

Table 10 Frequency parameters $\sqrt{\lambda}=\omega a^2\sqrt{\rho/D}$

Boundary conditions	Mode No.	Aspect ratio $\alpha=a/b$(Leissa/present)		
		0.4	1.0	2.5
SFSF	1.	9.7600/ 9.7737	9.6314/ 9.6388	9.4841/ 9.4914
	2.	11.0368/11.1321	16.1648/16.1350	33.6228/ 33.6229
	3.	15.0626/15.2951	36.7256/37.1786	38.3629/ 38.7143
	4.	21.7044/22.3887	38.9450/39.2614	75.2037/ 75.2041
	5.	31.1771/30.1157	46.7381/46.7483	86.9684/ 86.3923
	6.	39.2387/40.8327	70.7401/67.0440	130.3576/350.8936
CFCF	1.	22.346 /22.382	22.272 /22.307	22.130 / 22.209
	2.	23.086 /23.031	26.529 /26.442	41.689 / 41.685
	3.	25.666 /24.944	43.664 /44.221	61.002 / 62.096
	4.	30.633 /31.169	61.466 /62.668	92.384 / 92.496
	5.	38.687 /40.111	67.549 /67.254	119.88 /126.19
	6.	49.858 /50.484	79.904 /82.158	157.76 /158.11

Table 11 Frequency parameters $\sqrt{\lambda}=\omega a^2\sqrt{\rho/D}$

Boundary conditions	Mode No.	Aspect ratio $\alpha=a/b$(Leissa/present)		
		0.4	1.0	2.5
CFFF	1.	3.5107/ 3.5273	3.4917/ 3.5245	3.4562/ 3.5233
	2.	4.7861/ 4.7746	8.5246/ 8.5443	17.988 / 17.987
	3.	8.1146/ 8.1870	21.429 /22.029	21.563 / 22.019
	4.	13.882 /14.248	27.331 /26.963	57.458 / 57.515
	5.	21.638 /21.423	31.111 /31.216	60.581 / 60.912
	6.	23.731 /23.654	54.443 /55.643	106.54 /106.13
SFFF	1.	2.6922/ 2.6865	6.6480/ 6.6362	14.939 / 14.817
	2.	6.5029/ 6.4276	15.023 /15.840	16.242 / 16.217
	3.	12.637 /12.976	25.492 /25.365	48.844 / 48.932
	4.	15.337 /15.420	26.926 /25.980	52.089 / 51.930
	5.	17.510 /17.863	48.711 /50.389	97.225 / 96.676
	6.	21.699 /20.132	50.849 /50.923	102.34 /102.26
FFFF	1.	3.4629/ 3.4312	13.489 /13.540	21.643 / 21.492
	2.	5.2881/ 5.1684	19.789 /19.284	33.050 / 32.960
	3.	9.6220/ 9.8751	24.432 /23.937	60.137 / 60.542
	4.	11.437 /12.125	35.024 /35.727	71.484 / 71.635
	5.	18.793 /19.236	35.024 /35.727	117.45 /118.15
	6.	19.100 /20.042	61.526 /62.138	119.38 /119.74

polynomials proposed in this study could be utilized very conveniently to solve vibration problems of plates. The present method is further considered to be applicable to plates with arbitrary shape(such as folded plate and corrugated plate) of which vibration analysis have been difficult due to the complexity of beam funtions.

4. CONCLUSIONS

In this study, orthogonal polynomials obtained by the Gram-Schmidt process were used as displacement functions. The present method enabled to overcome complexity of computer caculations incurred by using conventional beam functions. Computed results demonstrated an accuracy of less than 3%, compared with those obtained by other functions. This new displacemet function can be used not only to obtain baseline data of dynamic problems by anâlyzing all boundary conditions, but also to approach vibration analysis of plates with arbitrary shape.

REFERENCES

Abramowitz, M. and Cahill, W.F., 1955, "On the Vibration of a Square Clamped Plate", Journal of Assoc. Comput. Mach., Vol. 2, pp. 162~168.

Cheung, Y.K. and Cheung, M.S., 1971, "Flexural Vibration of Rectangular and Other Rolygonal Plates", Journal of ASCE, Vol. 97, EM 2, pp. 391~411.

Dickinson, S.M., 1978, "On the Use of Simply Supported Plate Function in Rayleigh's Method Applied to the Flexural Vibration of Rectangular Plates", Journal of Sound and Vibration, Vol. 59, pp. 143~146.

Dickinson, S.M. and Li, E.K.H., 1982, "On the Use of Simply Supported of Plate Functions in the Rayleigh-Ritz Method Applied to the Flexural Vibration of Rectangular Plates", Journal of Sound and Vibration, Vol. 80, pp. 292~297.

Hearmon, R.F.S., 1959, "The Frequency of Flexural Vibration of Rectangular Orthotropic Plates with Clamped or Supported Edges", Journal of Applied Mechanics, Vol. 26, pp. 537~540.

Hidaka, K., 1951, "Vibration of a Square Plate Clamped at Four Edges", Math. Jap., Vol. 2, pp. 97~101.

Hooker, R.J. and O'Briein, D.J., 1974, "Natural Frequencies of Box-Type Structures by a Finite Element Method", Journal of Applied Mechanics, Vol. 23, pp. 363~365.

Leissa, A.W., 1973, "The Free Vibration of Rectangular Plates", Journal of Sound and Vibration, Vol. 31, pp. 257~293.

Mizusawa, S.T., 1986, "Natural Frequencies of Rectangular Plates with Free Edge", Journal of Sound and Vibration, Vol. 105, pp. 451~549.

Munakata, K., 1952, "On the Vibration and Elastic Stability of a Rectangular Plate Clamped at Its Four Edges", Jornal of Math. and Phys., Vol. 31, pp. 69~74.

Nishimura, T., 1953, "Studies on Vibration Problems of Flat Plates by Means of Difference Calculus", Proc. 3d Jap. Natl. congr. Appl. Mech., Vol. 6, pp. 417~420.

Stanisic, M., 1957, "An approximate Method Applied to the Solution of the Problem of Vibrating Rectangular Plates", Journal of Aeron. Sci., Vol. 24, pp. 159~160.

Warburton, G.B., 1954, "The Vibration of Rectangular Plates", Proceeding of the Institution of Mechanical Engineerns, Vol. 168, pp. 317~384.

Young, D., 1950, "Vibration of Rectangular Plates by the Ritz Method", Journal of Applied Mechanics, Vol. 17, pp. 448~453.